Program : **B.Tech**

Subject Name: **Engineering Physics**

1.○What do you understand by matter waves? Or Discuss the concept of de-Broglie matter waves.
2.○Define group velocity and particle velocity in reference of a group wave. Or Define particle velocity, group velocity and phase velocity.
3.○Derive mathematical expression for group and phase velocity.
4.○Show that the phase velocity of matter waves exceeds the velocity of light.
5.○Give the relation between group velocity and particle velocity. Or Shoe that the group velocity of matter waves equals particle velocity. Or Establish relation between particle and group velocities.
6.○Show that for non-relativistic free particle the phase velocity is half of the group velocity.
7.○Show that group velocity is less than the phase velocity in dispersive medium.
8.○Explain Heisenberg uncertainty principle with an example. Or Prove that electrons do not exist inside the nucleus using uncertainty principle.
9.○Explain Heisenberg uncertainty principle with an example. Or Derive Heisenberg principle from hypothetical gamma ray microscope.
10.○Explain the diffraction of electron by a single slit to illustrate Heisenberg uncertainty principle.
11.○Explain Heisenberg uncertainty principle and give elementary proof for it.
12.○What do you understand by wave function, discuss its physical significance?
13.○What is meant by operators? Obtain an operator for the energy “E” and momentum “P”.
14.○Derive Schrödinger’s time independent wave equation.
15.○Derive Schrödinger’s time dependent wave equation.
16.○Obtain energy level for a particle trapped in infinitely deep square potential well.

UNIT-1
QUANTUM MECHANICS

Q.1 What do you understand by matter waves? **(Feb10)**
Or
Discuss the concept of de-Broglie matter waves. **(June 15)**

Ans: The optical phenomenon, such as interference, diffraction and polarization of light could be explained by wave theory of light; whereas photoelectric effect or Compton Effect of light could only be explained if we consider the light as particle. Hence light shows itself wave nature at one end while particle nature on the other hand.

This nature of light is known as dual nature and the property is known as wave particle duality.

In 1924 Louis de-Broglie proposed that the matter also possess dual character like light. His concept about the dual nature of matter was based on the following facts:

(i) Matter and light both are forms of energy and each of them can be transformed into the other.

(ii) Both are governed by the space time symmetries of the theory of the relativity.

According to Louis de-Broglie, a moving particle is surrounded by a wave whose wavelength depends upon the mass of the particle and its velocity. These waves associated with the matter particles are known as matter waves or de-Broglie waves.

De-Broglie provided a connection between, the wavelength of matter waves and momentum of the particle i.e. $\lambda = \frac{h}{p}$ (1)

Properties of Matter-waves

1. Matter-waves are associated with any moving body and their wavelength is given by $\lambda = \frac{h}{mv}$
2. The wavelength of matter-waves is inversely proportional to the velocity of the body. Hence, a body at rest has an infinite wavelength whereas the one traveling with a high velocity has a lower wavelength.
3. Wavelength of matter-waves depends on the mass of the body and decreases with increase in mass. Because of this, the wave-like behavior of heavier objects is not very evident whereas the wave nature of subatomic particles can be observed experimentally.
4. Amplitude of the matter-waves at a particular space and time depends on the probability of finding the particle at that space and time.
5. Unlike other waves, there is no physical quantity that varies periodically in the case of matter waves.
6. Matter waves are represented by a wave packet made up of a group of waves of slightly differing wavelengths. Hence, we talk of group velocity of matter waves rather than the phase velocity.
7. Matter-waves show similar properties as other waves such as interference and diffraction.

Q.2 Define group velocity and particle velocity in reference of a group wave. **(Dec 10)**
Or
Define particle velocity, group velocity and phase velocity. **(Dec 11,14, 15)**

Ans: **Group velocity:** When two or more than two waves travel in a medium then the superposition of these waves result in the formation of a wave packet. The velocity with which a wave packet advances in medium is called group velocity. The group velocity of the wave packet is given by-

$$v_g = \frac{d\omega}{dk}$$

Phase velocity or wave velocity: - when a monochromatic wave (wave of a single frequency or wavelength) travels through a medium, the velocity with which the wave advances in the forward direction of the medium is called wave or phase velocity. The phase velocity of the matter wave is given by- $v_p = \frac{\omega}{k} = \frac{2\pi v}{\frac{2\pi}{\lambda}} = \nu\lambda$

Particle velocity: Particle velocity is the velocity of the material particle moving in medium or space for a matter wave group velocity equals particle velocity. It is denoted by v.

Q.3 Derive mathematical expression for group and phase velocity. **(Dec 15)**

Ans: *(Include the definitions of phase and group velocities as above)*

Expression for group velocity: - When two or more than two waves travel in a medium then the superposition of these waves result in the formation of a wave packet. The velocity with which a wave packet advances in medium is called group velocity. Let us consider a wave group which consists of two harmonic waves of equal amplitude but slightly different frequencies ω_1 & ω_2 and propagation constants k_1 & k_2. Their separate displacements are given by-

$y_1 = a\sin(\omega_1 t - k_1 x)$ (1)

And $y_2 = a\sin(\omega_2 t - k_2 x)$ (2)

Their superposition gives-

$$y = y_1 + y_2$$

Or

$$Y = a[\sin(\omega_1 t - k_1 x) + (\omega_2 t - k_2 x)]$$

$$Y = 2\,a\sin\left[\frac{\omega_1 + \omega_2}{2}t - \frac{k_1 + k_2}{2}x\right]\cos\left[\frac{\omega_1 - \omega_2}{2}t - \frac{k_1 - k_2}{2}x\right]$$

As $\omega_1 \sim \omega_2 \sim \omega$ *and* $k_1 \sim k_2 \sim k$

Therefore $\frac{\omega_1+\omega_2}{2} = \omega$ and $\frac{k_1+k_2}{2} = k,$ $\omega_1 - \omega_2 = \Delta\omega$ *and* $k_1 - k_2 = \Delta k$

Hence $Y = 2a\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right]\sin[\omega t - kx]$ (3)

This represents a wave system with frequency ω and propagation constant k but with amplitude given by

$A = 2a\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right]$ (4)

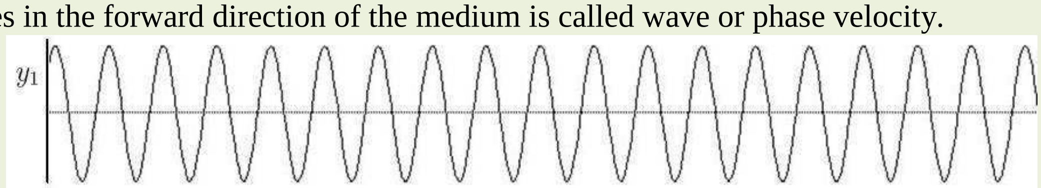
The wave system of the equation 3 can be represented as

The velocity of above wave system is called group velocity and represented by

$$v_g = \frac{\Delta\omega}{\Delta k}$$

If the frequency interval between the harmonics of the wave packet is infinitely small then $v_g = \frac{d\omega}{dk}$ (5)

Expression for Phase velocity or wave velocity: - when a monochromatic wave (wave of a single frequency or wavelength) travels through a medium, the velocity with which the wave advances in the forward direction of the medium is called wave or phase velocity.



A plane harmonic wave travelling along +ve x-direction is given by-

$$y = a\sin(\omega t - kx) \qquad (1)$$

Where 'a' is amplitude, $\omega(= 2\pi\nu)$ is the angular frequency and $k(= \frac{2\pi}{\lambda})$ is the propagation constant of the wave.

The quantity $(\omega t - kx)$ in the wave equation is called the phase of the wave. The planes of constant phase are defined by-

$$(\omega t - kx) = constant$$

Differentiating above wrt time, we get-

$$\omega - k\frac{dx}{dt} = 0 \qquad (2)$$

Or $$\frac{dx}{dt} = v_p = \frac{\omega}{k} = \frac{2\pi\nu}{\frac{2\pi}{\lambda}} = \nu\lambda \qquad (3)$$

Thus velocity with which planes of constant phase advances through the medium is equal to the wave velocity.

Q.4 Show that the phase velocity of matter waves exceeds the velocity of light.

Ans: The velocity with which planes of constant phase advances through the medium equal to the wave velocity or phase velocity and is given as

$$v_p = \frac{\omega}{k} = \frac{2\pi\nu}{\frac{2\pi}{\lambda}} = \nu\lambda$$

As we know that $$\omega = 2\pi\nu = \frac{2\pi m c^2}{h}$$

By the theory of special relativity $m = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$

Therefore $$\omega = \frac{2\pi m_0 c^2}{h\sqrt{1-\frac{v^2}{c^2}}}$$ And $$k = \frac{2\pi}{\lambda} = \frac{2\pi m v}{h}$$

$$k = \frac{2\pi m_0 v}{h\sqrt{1-\frac{v^2}{c^2}}}$$

As $v_p = \frac{\omega}{k}$ Or $$v_p = \frac{\frac{2\pi m_0 c^2}{h\sqrt{1-\frac{v^2}{c^2}}}}{\frac{2\pi m_0 v}{h\sqrt{1-\frac{v^2}{c^2}}}}$$ Or $$v_p = \frac{c^2}{v}$$

Since no material particle can travel faster than the velocity of light, therefore phase velocity of matter wave is greater than the velocity of light.

Q.5 Give the relation between group velocity and particle velocity. **(Dec 10, 14, 15, 17)**

Or

Shoe that the group velocity of matter waves equals particle velocity.

Or

Establish relation between particle and group velocities.

Ans: **Proof group velocity equals particle velocity-** Group velocity of a matter wave is given as

$$v_g = \frac{d\omega}{dk} \quad \text{or} \quad v_g = \frac{\frac{d\omega}{dv}}{\frac{dk}{dv}}$$

$$\frac{d\omega}{dv} = \frac{d}{dv}\left[\frac{2\pi m_0 c^2}{h}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}\right]$$

	$\frac{d\omega}{dv} = \frac{2\pi m_0 c^2}{h}\left[-\frac{1}{2}\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}\left(-\frac{2v}{c^2}\right)\right]$ $\frac{d\omega}{dv} = \frac{2\pi m_0 v}{h}\left[\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}\right]$ Now $\frac{dk}{dv} = \frac{d}{dv}\left[\frac{2\pi m_0 v}{h}\ (1-\frac{v^2}{c^2})^{-\frac{1}{2}}\right]$ Or $\frac{dk}{dv} = \left[\frac{2\pi m_0}{h}\left\{v\left(-\frac{1}{2}\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}\left(-\frac{2v}{c^2}\right)\right)+(1-\frac{v^2}{c^2})^{-\frac{1}{2}}\right\}\right]$ $\frac{dk}{dv} = \left[\frac{2\pi m_0}{h}\left\{\left(\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}\left(\frac{v^2}{c^2}\right)\right)+(1-\frac{v^2}{c^2})^{-\frac{1}{2}}\right\}\right]$ $\frac{dk}{dv} = \left[\frac{2\pi m_0}{h}(1-\frac{v^2}{c^2})^{-\frac{3}{2}}\left\{\left(\frac{v^2}{c^2}\right)+\left(1-\frac{v^2}{c^2}\right)\right\}\right]$ $\frac{dk}{dv} = \left[\frac{2\pi m_0}{h}(1-\frac{v^2}{c^2})^{-\frac{3}{2}}\right]$ Therefore $v_g = \frac{\frac{2\pi m_0 v}{h}\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}}{\frac{2\pi m_0}{h}(1-\frac{v^2}{c^2})^{-\frac{3}{2}}}$ or $v_g = v$
Q.6	Show that for non-relativistic free particle the phase velocity is half of the group velocity.**(Jan 16, Jun 13 Dec 15)**
Ans:	As we know that for a non relativistic free particle its total energy is $E = \frac{1}{2}mv^2$ also $v_g = v$ Therefore $E = \frac{1}{2}mv_g^2$ as $E = h\nu$ Hence $h\nu = \frac{1}{2}mv_g^2$ For phase velocity of matter waves $v_p = \nu\lambda$ or $h\frac{v_p}{\lambda} = \frac{1}{2}mv_g^2$ According to de-Broglie's hypothesis $\lambda = \frac{h}{mv}$ or $\lambda = \frac{h}{mv_g}$ Therefore $v_p = \frac{1}{2}v_g$
Q.7	Show that group velocity is less than the phase velocity in dispersive medium. **(Jun 14)**
Ans:	**Relation between Phase and Group velocity:** The phase velocity of a wave is given by $v_p = \frac{\omega}{k}$ and $v_g = \frac{d\omega}{dk}$

$$v_g = \frac{d}{dk}(kv_p)$$

Or $$v_g = kv_p + k\frac{dv_p}{dk}$$

As $$k = \frac{2\pi}{\lambda}$$

$$v_g = kv_p + \frac{2\pi}{\lambda}\frac{dv_p}{d(\frac{2\pi}{\lambda})}$$

$$v_g = kv_p + \frac{1}{\lambda}\frac{dv_p}{d(\frac{1}{\lambda})}$$

Therefore $$v_g = v_p - \lambda\frac{dv_p}{d\lambda}$$

This is the desired relation between group velocity $\boldsymbol{v_g}$ phase velocities $\boldsymbol{v_p}$ **.**

Q.8 Explain Heisenberg uncertainty principle with an example. **(Jun 10,12 16, Dec 12)**

Or

Prove that electrons do not exist inside the nucleus using uncertainty principle. **(Apr 10)**

Ans: **Heisenberg's Uncertainty Principle:** According to Heisenberg's uncertainty principle "it is impossible to determine simultaneously the position and momentum of any particle with any desired accuracy, rather the product of uncertainties in the position and momentum is always greater than or equal to h/4π, where h is Planck's constant.

Mathematically $\Delta x\Delta p \geq \frac{h}{4\pi}$ Where Δx Uncertainty in the position

Δp Uncertainty in the momentum

Like position and momentum, other two pairs of conjugate physical quantities are *time* and *energy*, and orbital angular momentum and angular position, the Heisenberg's uncertainty principle follows for these two pairs also and can be written as $\Delta E\Delta t \geq \frac{h}{4\pi}$ and $\Delta L\Delta\theta \geq \frac{h}{4\pi}$

Where ΔE uncertainty in the Energy

Δt Uncertainty in the time

ΔL Uncertainty in the orbital angular momentum

$\Delta\theta$ Uncertainty in the angular position

Non Existence of Electrons in the Nucleus

The size of a typical nucleus is of the order of 10^{-14}m. If any particle is to exist within nucleus then the uncertainty in the position of the particle will be-

$$\Delta x = 2\ X\ 10^{-14}m$$

Then according the uncertainty principle the uncertainty in the momentum will be given as

$$\Delta p = \frac{h}{2\ X\ 10^{-14}m} = \frac{6.6\ X\ 10^{-34}}{2\ X\ 10^{-14}m} = 3.31\ X\ 10^{-20}\ Kg\ m/sec$$

Thus the magnitude of the momentum of the particle must be at least of this order. Then the relativistic energy of electron in the nucleus will be

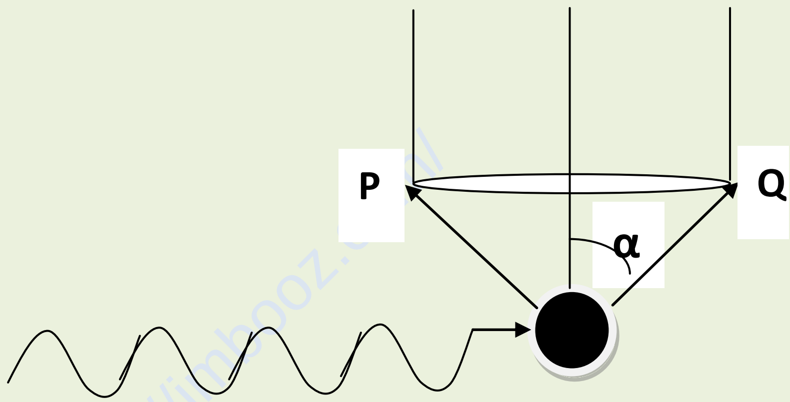
$$E = \sqrt{m_0^2c^4 + p^2c^2}$$

$$= \sqrt{(9.1\ X\ 10^{-31})^2 X\ (3\ X\ 10^8)^4 + (3.31\ X\ 10^{-20})^2 X\ (3\ X\ 10^8)^2}$$

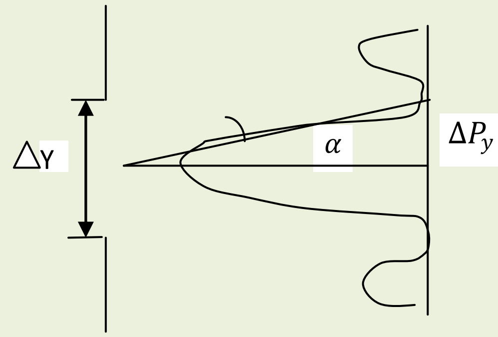
$$= \sqrt{(6.7\ X\ 10^{-27}) + (98.6\ X\ 10^{-24})}$$

$$= \sqrt{(9860 + 0.67) X\ 10^{-26}}$$

$$= 99.3\ X\ 10^{-13}\ Joules$$

	$= 62\ MeV.$ Thus if electron resides insides the nucleus, it must possess energy of the order of $62\ MeV$ while the electrons emitted in beta decay have only the energies of the order of $3\ MeV$. The big mismatch in two values confirms that electron does not reside inside the nucleus.
Q.9	Explain Heisenberg uncertainty principle with an example. **(Jun 10,12 16, Dec 12)** Or Derive Heisenberg principle from hypothetical gamma ray microscope. **(June 13)**
Ans.	**Determination of position of electron by gamma ray microscope** Consider a free electron beneath the center of the microscope's lens The circular lens forms a cone of angle 2α from the electron. The electron is then illuminated from the left by gamma rays--high energy light which has the shortest wavelength. According to a principle of wave optics, the microscope can resolve objects to a size of Δx, which is related to the wavelength λ of the gamma ray, by the expression: $$\Delta x = \lambda / (2\sin A)$$  To be observed by the microscope, the gamma ray must be scattered into any angle within the cone of angle 2α. In quantum mechanics, the gamma ray carries momentum, as if it were a particle. The total momentum p is related to the wavelength by the formula $p = h / \lambda$, where h is Planck's constant. In the extreme case of diffraction of the gamma ray to the right edge of the lens, i.e. at the point Q then the momentum in the x direction would be $(h/\lambda) \sin\alpha$, while at another extreme at point P the momentum in the x direction would be $(-h/\lambda) \sin\alpha$ The total change in the momentum of the photon will be $$\Delta p = (h/\lambda) \sin\alpha - (-(h/\lambda) \sin\alpha)$$ $$\Delta p = (2h/\lambda) \sin\alpha$$ $$\Delta x \Delta p = h$$ We obtain a reciprocal relationship between the minimum uncertainty in the measured position, Δx, of the electron along the x axis and the uncertainty in its momentum, Δp, in the x direction this is in accordance with the Heisenberg uncertainty principle.
Q.10	Explain the diffraction of electron by a single slit to illustrate Heisenberg uncertainty principle. **(Dec 14)** Or Explain Heisenberg uncertainty principle with an example. **(Jun 10,12 16, Dec 12)**
Ans:	**Diffraction of electron by single slit** Let us consider a narrow beam of electrons of momentum *p* is travelling in *+x* direction and

passing through a slit of width Δy. Since electrons are passing through the width Δy therefore the uncertainty in the position of the electrons will be equal to Δy . A diffraction pattern as shown in the figure will be formed by electron beam after passing through the slit.



The first minimum of Fraunhoffer diffraction is given as $d\sin\alpha = n\lambda$. For the first order minima $n = 1$. The slit width is Δy

therefore $\Delta y \sin\alpha = \lambda$ or $\Delta y = \frac{\lambda}{\sin\alpha}$ 1

The moving electrons initially have no component of momentum along y- direction because they are moving along x-axis. But after diffraction from slit the electrons have a component of momentum $p\sin\alpha$ along y-direction. Now as electron end up anywhere between $-\alpha$ to α, the y-component of momentum may lie somewhere between $p\sin\alpha$ to $-p\sin\alpha$. Hence the uncertainty in the y-component of the momentum will be

$\Delta p = p\sin\alpha - (-p\sin\alpha) = 2p\sin\alpha$

Or $\Delta p = \frac{2h}{\lambda}\sin\alpha$ 2

Taking the product of equation 1 and 2 we get

$\Delta y . \Delta p = \frac{2h}{\lambda}\sin\alpha . \frac{\lambda}{\sin\alpha}$

$\Delta y . \Delta p = 2h$

The above relation is in agreement with the uncertainty principle.

Q.11 Explain Heisenberg uncertainty principle and give elementary proof for it. **(Dec 12)**

Ans: **Heisenberg's Uncertainty Principle:** According to Heisenberg's uncertainty principle "it is impossible to determine simultaneously the position and momentum of any particle with any desired accuracy, rather the product of uncertainties in the position and momentum is always greater than or equal to h/4π, where h is Planck's constant.

Mathematically $\Delta x \Delta p \geq \frac{h}{4\pi}$ Where Δx Uncertainty in the position

Δp Uncertainty in the momentum

Proof: It is possible to prove the Heisenberg's principle by using the fact that a moving particle is associated by a group of waves and the group velocity equals particle velocity. Since particle is considered as group of waves this implies that the particle cannot be considered as a localized entity. It indicates that there is always a limit to the accuracy with which one can measure its particle properties.

Let us consider a particle of mass *m* moving with the velocity *v*, the particle can be shown as surrounded by de-Broglie waves as shown in the figure.

Formation of above wave packet can be explained by considering two waves of angular

frequencies ω_1 and ω_2 and propagation constants k_1 and k_2.
These waves can be represented as

$$\psi_1 = A\sin(\omega_1 t - k_1 x) \qquad (1)$$

$$\psi_2 = A\sin(\omega_2 t - k_2 x) \qquad (2)$$

Superposition of these two results in

$$\psi = \psi_1 + \psi_2 \qquad (3)$$

$$\psi = A\sin(\omega_1 t - k_1 x) + A\sin(\omega_2 t - k_2 x)$$

As $\omega_1 \sim \omega_2 = \omega$ and $k_1 \sim k_2 = k$ therefore $\frac{\omega_1+\omega_2}{2} = \omega$ and $\frac{k_1+k_2}{2} = k$ Also $\omega_1 - \omega_2 = \Delta\omega$ and $k_1 - k_2 = \Delta k$
Hence

$$\psi = 2A cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right) sin(\omega t - kx) \qquad (4)$$

The resultant wave packet travels with velocity v_g. The position of the particle is not certain rather it is somewhere between one node and next node. The error in the measurement of the position of the particle is therefore equal to the distance between these two nodes.

A node is formed when $cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right) = 0$.

This is possible when $\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots..$ thus if x_1 and x_2 represents the position of the two successive nodes, then at any instant t, we get

$$\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x_1\right) = (2n+1)\frac{\pi}{2} \qquad (5)$$

$$\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x_2\right) = (2n+3)\frac{\pi}{2} \qquad (6)$$

On subtracting equation 5 from equation 6 we get

$$\frac{\delta k}{2}(x_2 - x_2) = \pi$$

$$x_2 - x_1 = \frac{2\pi}{\delta k} \text{ therefore } \Delta x = \frac{2\pi}{\Delta k} = \frac{2\pi}{\Delta\lambda} = \Delta\lambda$$

$$\Delta x = \frac{h}{\Delta p} \text{ or } \Delta x\Delta p = h$$

More precise calculations show that $\Delta x\Delta p \geq \frac{h}{4\pi}$

Energy & time uncertainty relation

As we know $t = \frac{x}{v_g}$ and $\Delta t = \frac{\Delta x}{v_g}$ (8)

also $E = \frac{1}{2}mv^2$ or $2E = mv^2$ on differentiating wrt to v we get

$$\frac{dE}{dv} = 2mv \; or \; dE = mvdv \text{ or } \Delta E = mv\Delta v \qquad (9)$$

Using equation 8 and 9 we get

$$\Delta E\Delta t \geq \frac{\Delta x}{v_g} mv\Delta v \; or$$

$$\Delta E\Delta t \geq \Delta x\Delta p$$

Therefore $\Delta E\Delta t = h$

Q.12 What do you understand by wavefunction, discuss its physical significance? **(Dec14)**

Or

Discuss the concept of wavefunction associated with particle. Give examples of admissible wave function. Why derivative of wave function should be continuous everywhere? **(Jun 13)**

Or

Discuss the concept of probability density. Also define the wave function ϕ . **(Jun 15)**

Ans:	Variations of the 'Ψ' forms the matter waves. So it converts the particle and its associated wave statistically. The wave function or complex displacement Ψ is a complex quantity and we cannot measure it. The matter wave can be represented by wave function. This wave function is used to identify the state of a particle in an atomic structure. It tells us where the particle is likely to be not where it is. The probability of finding a particle in a particular volume element dτ is given by, $P(r)\, d\tau = \lvert \Psi^* \Psi \rvert \, d\tau$ Where Ψ^* is called the complex conjugate of Ψ. Being a complex function, it does not have a direct physical meaning, but when we multiply this with its complex conjugate, the product $\lvert \Psi \rvert^2$ has physical meaning. [We will speak normally the intensity of light at a point rather than the amplitude of light at a point since intensity (square of amplitude) is a measurable and real quantity]. A physically acceptable wave function must possess the following properties: i) 'Ψ' must be single valued everywhere inside a wave packet. ii) 'Ψ' must be finite since it tells us about the probability. iii) 'Ψ' must be continuous i.e. the derivative of the 'Ψ' should not vanish at the boundaries of wave packet. As 'Ψ' is related to a real particle, it cannot have a discontinuity at any boundary where potential changes. iv) $\iiint \lvert \Psi \rvert^2 d\tau = 1$ when the particle presence is certain in the space. Ψ Satisfying above requirement is said to be normalized.
Q.13	What is meant by operators? Obtain an operator for the energy "E" and momentum "P". **(Dec 14, 15)**
Ans:	In quantum mechanics operators are the mathematical operations, corresponding to physical observables. When these operators operate on a wave function then they give the Eigen value of corresponding physical observable. **Operator of energy** Let us consider a matter wave travelling in the positive *x* direction it could be represented by $\psi(x,t) = Ae^{-i(\omega t - kx)}$ 1 Differentiating equation (1) with respect to time $\frac{\partial \psi}{\partial t} = -i\omega A e^{-i(\omega t - kx)}$ As $\omega = 2\pi\nu$ and $E = h\nu$ Therefore $\frac{\partial \psi}{\partial t} = -i\frac{2\pi E}{h} A e^{-i(\omega t - kx)}$

$$\frac{\partial \psi}{\partial t} = -i\frac{2\pi E}{h}\psi$$

$$i\frac{h}{2\pi}\frac{\partial \psi}{\partial t} = E\psi$$

Or

$$E\psi = i\hbar\frac{\partial \psi}{\partial t}$$

Therefore $E = i\hbar\frac{\partial}{\partial t}$ is called operator of energy

Operator of momentum

Let us consider a matter wave travelling in the positive *x* direction it could be represented by

$\psi(x,t) = Ae^{-i(\omega t - kx)}$ 1

Differentiating equation (1) with respect to space

$$\frac{\partial \psi}{\partial x} = ikAe^{-i(\omega t - kx)}$$

As $k = \frac{2\pi}{\lambda}$ and $\lambda = \frac{h}{mv} = \frac{h}{P}$

Therefore

$$\frac{\partial \psi}{\partial x} = i\frac{2\pi P}{h}Ae^{-i(\omega t - kx)}$$

$$\frac{\partial \psi}{\partial x} = i\frac{2\pi P}{h}\psi$$

$$-i\frac{h}{2\pi}\frac{\partial \psi}{\partial x} = P\psi$$

Or

$$P\psi = -i\hbar\frac{\partial \psi}{\partial x}$$

Therefore $P = -i\hbar\frac{\partial}{\partial x}$ is called operator of momentum.

Q.14 Derive Schrodinger's time independent wave equation. **(Feb 10, Dec 11, 12, 13, 14 Apr 10, Jun 15, Dec 17)**

Ans: A matter wave travelling in the positive *x* direction with angular frequency ***'ω'*** and wave number ***'k'*** can be represented by

$$\psi(x,t) = Ae^{-i(\omega t - kx)} \quad (1)$$

Differentiating equation (1) with respect to space

$$\frac{\partial \psi}{\partial x} = ikAe^{-i(\omega t - kx)}$$

Differentiating again with respect to space

$$\frac{\partial^2 \psi}{\partial x^2} = i^2k^2Ae^{-i(\omega t - kx)} \quad (2)$$

	As $k = \frac{2\pi}{\lambda}$ and $\lambda = \frac{h}{mv}$ Therefore equation (2) can be written as $$\frac{\partial^2\psi}{\partial x^2} = i^2\frac{4\pi^2m^2v^2}{h^2}\psi$$ Or $$\frac{\partial^2\psi}{\partial x^2} + \frac{4\pi^2m^2v^2}{h^2}\psi = 0 \qquad (3)$$ Total energy of a particle can be written as $$E = V + \frac{1}{2}mv^2$$ Or $$2m(E - V) = m^2v^2 \qquad (4)$$ By using equations (3) and (4) we can write $$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2m}{h^2}(E - V)\psi = 0$$ The equation above is known as time independent wave equation it can also be written as $$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$
Q.15	Derive Schrodinger's time dependent wave equation.
Ans:	A matter wave travelling in the positive x direction with angular frequency ***'ω'*** and wave number ***'k'*** can be represented by $\psi(x,t) = Ae^{-i(\omega t - kx)}$ (1) The time independent wave equation for particle above can be written as $$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$ Rearranging the terms in equation above $$\frac{\partial^2\psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E - V)\psi$$ $$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi - V\psi$$ $$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi$$ Using the operator of energy equation above can be written as: $$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$ Above form of the equation is known as time dependent Schrodinger wave equation.

Q.16	Obtain energy level for a particle trapped in infinitely deep square potential well. **(Jun 14)** **Or** Find the energy Eigen values and corresponding de-Broglie’s wavelength associated with the lowest three energy state of particle enclosed in one dimensional infinite potential well. **(Jun 13)** **Or** Obtain expression of energy levels for particle trapped in one dimensional square with infinitely deep potential well. **(Dec 13)** **Or** Obtain expression for Eigen function of particle in one dimensional potential well of infinite height. **(Dec 11)** **OR** Obtain wave function expression for a particle trapped in infinitely deep square potential well. **(Jun 14)**
Ans:	**Particle in One Dimensional Box** Let us consider a particle moving inside a box. The dimension of box is ‘L’. The description of potential inside the box ix $V = 0 \; for \; 0 < x < L$ $V = \infty \; for \; x \leq 0 \; and \; x \geq L$ Since potential has infinite value at $x \leq 0 \; and \; x \geq L$ i.e. particle cannot exist on the boundary of the box and also outside of the box. Therefore the waves function $\psi = 0$ at the boundary of the box. Since the $V = 0$ inside the box therefore the time independent Schrödinger equation inside the box will be $\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E\psi = 0$ 1 Or $\frac{\partial^2 \psi}{\partial x^2} + K^2\psi = 0$ 2 Where $K = \sqrt{\frac{2mE}{\hbar}}$ The general solution for the above equation will be $\psi = A \sin Kx + B \cos Kx$ 3 Using boundary conditions $\psi = 0$ at $x = 0$ we get $0 = A \sin 0 + B$ or $B = 0$ Equation 3 now becomes $\psi = A \sin Kx$ 4 $\psi = 0$ At $x = L$ therefore equation 4 now is $0 = A \sin KL$ Since $A \neq 0$ Therefore $0 = \sin KL$ or $KL = n\pi$ here *n=1, 2, 3….* Now equation 4 can be rewritten as $\psi = A \sin \frac{n\pi}{L} x$ 5 The energy Eigen value will be $E = \frac{K^2\hbar^2}{2m}$ Or $E = \frac{n^2\pi^2\hbar^2}{2mL^2}$ 6

	From the equation above it is evident that the energy levels of the particle inside a box are quantized. **Normalized Wave Function:** On applying the condition of normalization over the wave function $\int_0^L \lvert\psi\rvert^2 dx = 1$ $A^2 \int_0^L sin^2\left(\frac{n\pi x}{L}\right) dx = 1$ Or $\frac{A^2}{2}\int_0^L \left[1 - \cos\left(\frac{2\pi n x}{L}\right)\right] dx = 1$ $\Longrightarrow$ $\frac{A^2}{2}[L] = 1$ Or $A = \sqrt{\frac{2}{L}}$ By placing the value of A in equation 5 we get normalized wave function/Eigen function. $\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x$
Q.17	Obtain energy level for a free particle. Or Obtain the solution of Schrodinger wave equation for a free particle.
Ans:	The time independent wave equation for particle above can be written as $\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$ (1) For a free particle value of V can conveniently be considered as zero, hence the time independent wave equation for a free particle will be $\frac{\partial^2\psi}{\partial x^2} + \frac{2mE}{\hbar^2}\psi = 0$ Or $\frac{\partial^2\psi}{\partial x^2} + K^2\psi = 0$ (2) Where $K = \sqrt{\frac{2mE}{\hbar}}$ (3) As the particle is free therefore no boundary condition can be applied on the general solution of the equation given by $\psi = A \sin Kx + B \cos Kx$ Therefore using equation (3) energy Eigen values of the free particle can be given as- $E = \frac{\hbar^2 K^2}{2m}$ (4) From equation above it can be seen that the energy levels of a free particle are continuous.